

Bending Stresses for Simple Shapes

In bending, the maximum stress and amount of deflection can be calculated in each of the following situations. Additional examples are available in an engineering handbook.

σ = maximum stress at outer fiber

Δ = deflection

P = force (load)

W = distributive load

L = length of beam

Z = section modulus

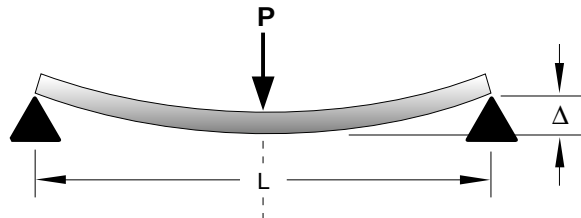
E = flexural modulus

I = moment of inertia about bending axis

Simply Supported Beam, Center Load

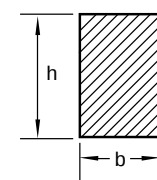
$$\sigma = \frac{PL}{4Z}$$

$$\Delta = \frac{PL^3}{48EI}$$



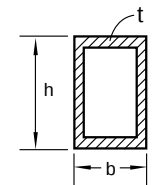
Section Modulus and Moment of Inertia

Section modulus and moment of inertia depend on the part's geometry and are listed for several common part geometries.



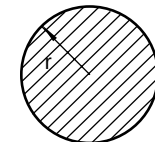
$$I = \frac{bh^3}{12}$$

$$Z = \frac{bh^2}{6}$$



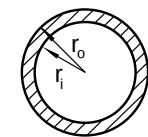
$$I = \frac{bh^3 - (b - 2t)(h - 2t)^3}{12}$$

$$Z = \frac{2I}{h}$$



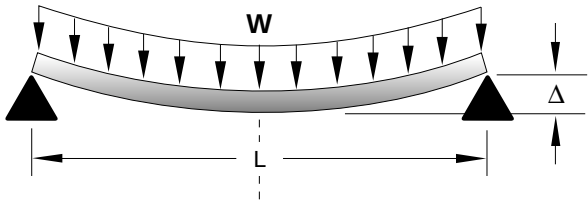
$$I = 0.78r^4$$

$$Z = 0.78r^3$$



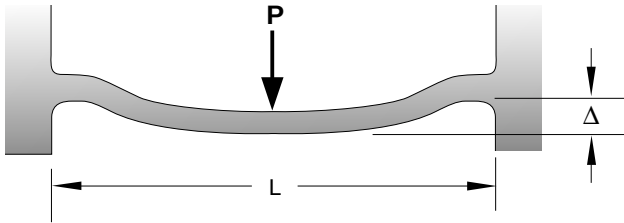
$$I = 0.78(r_o^4 - r_i^4)$$

$$Z = \frac{0.78(r_o^4 - r_i^4)}{r_o}$$



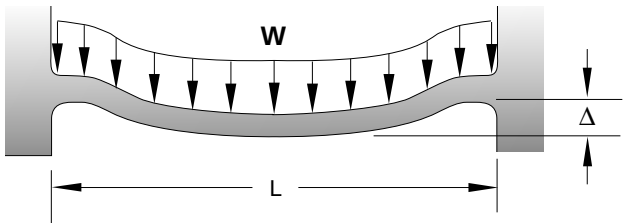
Simply Supported Beam, Uniform Load

$$\sigma = \frac{WL}{8Z} \quad \Delta = \frac{5WL^4}{384EI}$$



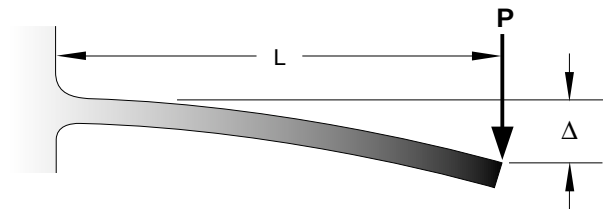
Fixed Supported Beam, Center Load

$$\sigma = \frac{PL}{8Z} \quad \Delta = \frac{PL^3}{192EI}$$



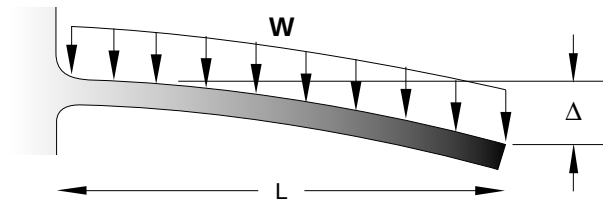
Fixed Supported Beam, Uniform Load

$$\sigma = \frac{WL^2}{12Z} \quad \Delta = \frac{WL^4}{384EI}$$



Cantilever Beam, End Load

$$\sigma = \frac{PL}{Z} \quad \Delta = \frac{PL^3}{3EI}$$



Cantilever Beam, Uniform Load

$$\sigma = \frac{WL^2}{2Z} \quad \Delta = \frac{WL^3}{8EI}$$

Finite Element Analysis (FEA)

Overview

FEA is a method for simulating component interaction within a given operating environment. FEA enables the designer to view several different design variations without doing extensive testing in the preliminary design stages. FEA can also be used to prevent overdesigning a part to ensure a “good” safety factor, and to reduce the number of design iterations to reach an optimum profile.

Types of Analysis

FEA allows the user to attempt many different design alternatives based on various loading conditions, including extended time of load, high-temperature and high moisture conditions. With these analyses, the designer can look at part optimization and examine how several part variations can make dramatic differences in part performance. Some of the forms of analysis are:

- Stress Analysis - used to calculate relative stress levels in components under static or dynamic loads, thermally applied stress, etc.
- Deflection Analysis - used to examine how a component will deform under various loading conditions.



Performing Finite Element Analysis

USE FEA WHEN

- *Designing complex shapes*
 - *Using unusual loading (i.e., impact, non-uniform)*
 - *Relative results are important (i.e., comparing several different designs)*

- Heat Transfer Analysis - used to show how the temperature will vary within a component for different boundary conditions and how a component will deform when there is a thermal differential across the component's surfaces.

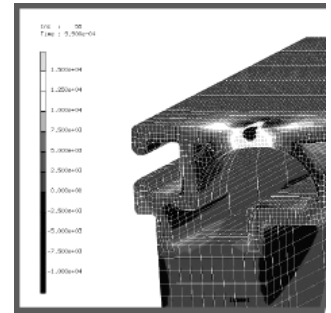
Procedure

The designer must provide the following input to the engineer performing the FEA:

1. A wireframe model that represents the component(s) being analyzed.
The model should have enough detail to accurately portray the critical areas of the component without attempting to model everything.
2. The physical properties of the material used in the design of the component.
3. The application of loads and restraints simulating the proposed operating conditions to be experienced by the component.

Interpretation of FEA results

Once the designer has properly input the parameters of the component(s) and the analysis has been run, it is necessary to interpret the results. In general terms, the results from an FEA analysis can be very accurate. It is the responsibility of the design engineer, however, to realize that FEA is simply a tool to aid in design.



Typical results from FEA

DESIGN TIP

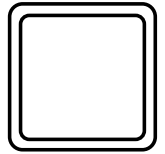
FEA results should be verified by proper testing.

Profile Symmetry

Symmetric profile geometry is generally easier to extrude than non-symmetric. Hollow profiles with no interior walls are also generally easy to extrude, regardless of symmetry. A symmetric profile produces a more uniform transition from the extruder to the entrance of the die. This results in a more uniform flow of material around the entire profile as it exits the die. Properly designed profiles not only increase extrusion speed, they lower residual stresses within a part and provide balanced flow. For more information on extrusion refer to [Section 3](#) of this guide. Different levels of symmetry for four profiles are illustrated at left.

Example 'A' is symmetric with no interior walls. This profile represents the ideal extrusion. A hollow pipe, square or round, is the easiest extrusion. The simple transition from the extruder to the die entrance allows for easy control of the material flow across the profile.

Example 'B' is a non-symmetric hollow profile with no interior walls. This profile is more difficult to extrude than profile A, but the lack of interior walls results in higher line speeds and lower residual stresses.



A - Easiest



B - Easy